Observability Analysis for Target Maneuver Estimation via Bearing-Only and Bearing-Rate-Only Measurements

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The implementation of modern guidance laws requires the design of tracking filters that provide information about the target maneuver. Because of the nonlinearity of the tracking problem, the stability of extended Kalman filters in tracking applications involving bearing-(rate-)only observations is highly dependent on the intercept geometry. This is due mainly to the lack of observability in the absence of range measurements. In this paper, the observability of target maneuvers via bearing-(rate-)only measurements is discussed. Intercept scenarios that result in the loss of observability are identified. They play an important role for both the initialization of tracking filters and the design of missile guidance laws based on bearing-(rate-)only measurements.

I. Introduction

In short-range air-to-air intercept scenarios, the most widely used guidance law is proportional navigation (PN). It is well known that the performance of PN is sharply reduced in the presence of target maneuvers. Improved guidance schemes make use of estimates of the current target acceleration and hence require a filter capable of tracking maneuvering targets.

The basic approach to the tracking problem is the design of an extended Kalman filter (EKF). If the observer (missile) is equipped with a passive seeker, bearing angles or bearing rates are the only measurements available about the missile-target relative motion. A tracking filter based on these measurements is often unstable because of the lack of observability. For nonmaneuvering targets, an observability analysis was carried out identifying geometries and observer maneuvers that result in unobservable filter states.[1-3] Although a considerable number of tracking filters for maneuvering targets have been proposed,[4-9] the observability of target maneuvers via bearing and bearing-rate-only measurements has never been discussed. Also, the dynamics of the target acceleration are usually modeled as uncorrelated noise processes in each direction of a Cartesian inertial reference system. However, the maneuvers of fixed-wing target aircraft are characterized by high lateral accelerations and low, often negligible axial accelerations. This information may be used to simplify the target model, thus reducing the uncertainty about the target motion.

The balance of this paper is divided into four sections. Section II discusses aspects of modeling. In Sec. III, the observability of target maneuvers is analyzed. Simulations demonstrating the main results of the observability analysis are presented in Sec. IV. A summary and conclusions are given in the last section.

II. System and Measurement Equations

The target tracking filter is based on a discrete EKF.[17,18] The derivation of the filter equations requires a mathematical model of the dynamics of the observer-target relative motion (system dynamics) as well as the measurement process. Because of the nonlinearity of the filtering problem, the behavior of the filter depends on the coordinates used to formulate the filter equations. Most frequently, Cartesian coordinates (see Fig. 1) with the state vector

$$z^T = (\Delta x, \Delta y, \Delta\dot{x}, \Delta\dot{y}, a_{Tx}, a_{Ty}) \tag{1}$$

have been used to derive the filter equations, where $(\ )^T$ denotes the transpose and $(\dot{\ })$ the total derivative with respect to time. The Cartesian coordinates render linear propagation equations for the filter states and covariance matrix if a linear model for the target acceleration dynamics is used.[6-8,10,11] However, the measurement equation is nonlinear if expressed in Cartesian coordinates, and it was shown that the resulting EKF is unstable for single sensor bearing-only measurements.[12,13] Several forms of polar coordinates were introduced that take advantage of the linearity of the measurement equation in a polar reference frame. The use of "modified polar coordinates" results in a decoupling of the observable and unobservable filter states in certain scenarios.[2] In another approach, Cartesian coordinates are used to solve the linear propagation equations, whereas the update is carried out in a polar reference frame.[14] In the following approach, the equations of the tracking filter will be based on ordinary polar coordinates because the use of Cartesian coordinates for both propagation and update may result in stability problems as mentioned earlier. Moreover, for the target model employed here, the system equations are nonlinear, even in a Cartesian reference frame. Hence, there is no advantage in using Cartesian coordinates for solving the propagation equations.

An important aspect of the modeling of the system dynamics is the dynamics of the target acceleration. Most of the tracking algorithms involving maneuvering targets are based on the assumption that the Cartesian components a_{Tx} and a_{Ty} are uncorrelated noise processes such as

$$\dot{a}_{Tx} = (a_{Tx} - w_x)/\tau \tag{2a}$$

$$\dot{a}_{Ty} = (a_{Ty} - w_y)/\tau \tag{2b}$$

where τ is the correlation time and w_x and w_y are uncorrelated Gaussian zero-mean white-noise processes.[6-8,11,14-16] However, the true target acceleration characteristics may be completely different. In fact, the components a_{Tx} and a_{Ty} may be highly correlated. For fixed-wing aircraft, the longitudinal acceleration component is often negligible compared to the lateral acceleration, especially in short-range intercept scenarios with strong evasive maneuvers. Therefore the constant-

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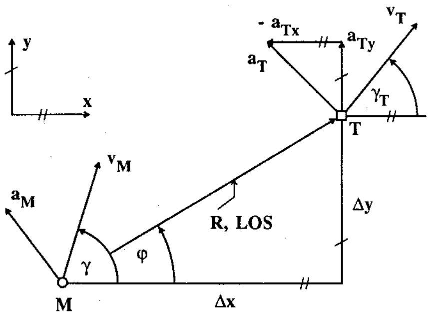

Fig. 1 Geometry of the planar tracking problem: *M*, missile; *v*, velocity; LOS, line of sight; φ, bearing angle; *T*, target; *a*, acceleration; *R*, range; γ, heading angle.

speed target model (CSTM) is used here to develop the filter equations. In the CSTM, the target is assumed to maneuver with lateral acceleration only. Hence, the target acceleration vector $\boldsymbol{a}_T$ is always directed normal to the velocity vector $\boldsymbol{v}_T$, as shown in Fig. 2. The dynamics of $\boldsymbol{a}_T$ are modeled in a way analogous to Eq. (2). Details will be given later. The advantage of the CSTM is the reduction of the uncertainty about the target motion by accounting for the correlation between a_{Tx} and a_{Ty}. Note, however, that using the CSTM renders the target dynamics nonlinear.

In the CSTM, the target velocity $\boldsymbol{v}_T$ is a parameter rather than a state of the tracking problem. For the subsequent analysis, $\boldsymbol{v}_T$ is assumed to be known (for example, from processing-bearing and range data prior to firing the missile). The target dynamics may therefore be expressed in terms of the heading angle γ_T (Fig. 2) as follows:

$$\dot{\gamma}_T = \boldsymbol{a}_T / \boldsymbol{v}_T \tag{3}$$

The polar acceleration components a_{TR} and $a_{T\varphi}$ are (see Fig. 2):

$$a_{TR} = -\boldsymbol{a}_T \sin(\gamma_T - \varphi) \tag{4a}$$

$$a_{T\varphi} = \boldsymbol{a}_T \cos(\gamma_T - \varphi) \tag{4b}$$

According to Figs. 1 and 2, the relative position of missile and target is given by the range R and the bearing angle φ with

$$\ddot{R} = a_{TR} - a_{MR} + R\dot{\varphi}^2 \tag{5a}$$

$$\ddot{\varphi} = (a_{T\varphi} - a_{M\varphi})/R - 2\dot{R}\dot{\varphi}/R \tag{5b}$$

where a_{MR} and $a_{M\varphi}$ denote the polar acceleration components of the missile. Selecting the polar state vector

$$\boldsymbol{y}^T = (\dot{\varphi}, \dot{R}, \varphi, R, \gamma_T, \dot{\gamma}_T) \tag{6}$$

and substituting Eqs. (3), (4) and (6) into Eq. (5) yields the following system equations:

$$\dot{y}_1 = \frac{v_T y_6 \cos(y_5 - y_3) - a_{M\varphi}}{y_4} - 2\frac{y_1 y_2}{y_4} \tag{7a}$$

$$\dot{y}_2 = -v_T y_6 \sin(y_5 - y_3) - a_{MR} + y_4 y_1^2 \tag{7b}$$

$$\dot{y}_3 = y_1 \tag{7c}$$

$$\dot{y}_4 = y_2 \tag{7d}$$

$$\dot{y}_5 = y_6 \tag{7e}$$

$$\dot{y}_6 = \frac{\dot{a}_T}{v_T} \tag{7f}$$

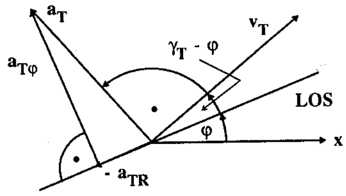

Fig. 2 Constant-speed target model.

The discrete version of the system dynamics is obtained by integrating Eq. (7) across the sampling interval

$$I_k = [t_k, t_{k+1}] \tag{8}$$

For this purpose, $\dot{a}_T$ in Eq. (7f) must be specified. Since the target behavior is unknown, it is assumed that

$$\dot{a}_T = 0 \tag{9}$$

The model [Eqs. (7) and (9)] is exact for constant target acceleration but produces propagation errors if the target acceleration is time-varying. In the latter case, an adaption scheme has to be used to stabilize the filter.[19] Following the approach taken in Ref. 2, the integration of Eqs. (7) and (9) may be carried out in closed form by integrating in a Cartesian reference frame and subsequently transforming the results to polar coordinates. Thus, one obtains the discrete equations of motion in the following form:

$$\boldsymbol{y}(k+1) = \boldsymbol{f}[\boldsymbol{y}(k), T] \tag{10}$$

where $T = t_{k+1} - t_k$ is the sampling period and $\boldsymbol{y}(k) \equiv \boldsymbol{y}(t_k)$.

The measurement equation is of the form

$$m(k) = C\boldsymbol{y}(k) + s(k) \tag{11a}$$

with

$$C = [1 \ 0 \ 0 \ 0 \ 0 \ 0] \tag{11b}$$

for bearing-rate-only measurements and

$$C = [0 \ 0 \ 1 \ 0 \ 0 \ 0] \tag{11c}$$

for bearing-only measurements; $s(k)$ is a Gaussian white-noise process with variance $S(k)$. The tracking filter is the EKF based on the system dynamics of Eq. (10) with the measurements of Eq. (11).

III. Observability Analysis

The observability analysis is concerned with the behavior of the filter covariance matrix $P \in R^{n \times n}$, where n is the system order; P represents the filter's assessment of the estimation accuracy. Let $P(i/j)$ denote the value of P at the time t_i based on the processing of the measurements $m(1),...,m(j)$. The goal of filtering is the reduction of the initial estimation uncertainty given by $P(0/0)$. The influence of the measurements on P becomes evident in the nonrecursive version of the covariance equations[17]:

$$P^{-1}(k/k) = W^T(0,k)P^{-1}(0/0)W(0,k) + I(k,0) \tag{12}$$

where

$$I(k,0) = \sum_{i=0}^{i=k} W^T(i,k)C^T S^{-1}(i) C W(i,k) \tag{13}$$

is the information matrix, and $W(i,k)$ is the $n \times n$ transition matrix associated with the system dynamics. In the nonlinear case, it is obtained by linearization of Eq. (10) around a nominal path $y^*(k)$:

$$W(i,j) = \frac{\partial f[y^*(j),T_{ij}]}{\partial y^*(j)} \tag{14}$$

with

$$T_{ij} = t_i - t_j \tag{15}$$

Assuming that the uncertainty about the initial state is infinite, i.e.,

$$P^{-1}(0/0) = 0 \tag{16}$$

it is obvious that $P(k/k)$ is bounded only if the information matrix is nonsingular. Otherwise, the filter diverges. In the following, scenarios (nominal paths y^*) that will result in a singular information matrix are sought. The analysis may be simplified by suppressing the explicit time dependency of $P(k/k)$, which is easily done by propagating Eq. (12) back to initial time.[2] The result is

$$P^{-1}(0/k) = P^{-1}(0/0) + W^T(k,0)I(k,0)W(k,0) \tag{17}$$

$P(0/k)$ is the error covariance matrix associated with $y(0/k)$, which is the estimated initial state based on k measurements. Since the initial time may be any time $t_j < t_k$, Eq. (17) is generalized to

$$P^{-1}(j/k) = P^{-1}(j/j) + \tilde{I}(k,j) \tag{18}$$

with

$$\tilde{I}(k,j) = \sum_{i=j}^{i=k} W^T(i,j)C^T S^{-1}(i) C W(i,j) \tag{19}$$

The states $y(j)$ and, hence, $y(k)$ are said to be completely observable with respect to the measurements $m(j),\dots,m(k)$ if and only if $\tilde{I}(k,j)$ is positive definite.

To investigate $\tilde{I}$ for the tracking problem, W from Eq. (14) and C from Eqs. (11) are substituted into Eq. (19). To handle both cases (11b) and (11c), the measurement equation is rewritten as

$$m(k) = y_q(k) + s(k) \tag{20}$$

and q is either 1 or 3. Substituting Eq. (20) into Eq. (19) results in

$$\tilde{I}(k,j) = \sum_{i=j}^{i=k} \frac{1}{S(i)} [w_{q1} w_{q2} \cdots w_{q6}]^T [w_{q1} w_{q2} \cdots w_{q6}] \tag{21}$$

where

$$w_{qr}(i,j) = \frac{\partial f_q[y^*(j),T_{ij}]}{\partial y_r^*(j)} \tag{22}$$

is the sensitivity of the measured state y_q with respect to changes in y_r. If w_{qr} vanishes for all i, the measurements contain no information about y_r. In this case, y_r is unobservable, and $\tilde{I}(k,j)$ becomes singular, as is evident from Eq. (21). To analyze the observability of target maneuvers, the sensitivities w_{15} and w_{16} associated with the target states y_5 and y_6 have to be investigated. The exact analytical expressions for w_{15} and w_{16} are given in Refs. 19 and 20. For the subsequent analysis, it suffices to consider their first-order approximation with respect to

$$\Delta\gamma_T = T_{ij} y_6(i) \tag{23}$$

The approximations are as follows[19,20]:

$$w_{15}(i,j) \approx -T_{ij} v_T y_6(j) \frac{z_1(i)}{y_4^2(i)} \left(\sin y_5(j) - \frac{z_2(i)}{z_1(i)} \cos y_5(j) \right) \tag{24}$$

$$w_{16}(i,j) \approx -T_{ij} v_T \beta_1 \sin y_5(j) + T_{ij} v_T \beta_2 \cos y_5(j) \tag{25}$$

with

$$\beta_1 = y_{11} T_{ij} + y_{13} + T_{ij} y_{14} y_6(j) \tag{26a}$$

$$\beta_2 = y_{12} T_{ij} + y_{14} + T_{ij} y_{13} y_6(j) \tag{26b}$$

and

$$y_{11} = \frac{z_4(i) - 2z_1(i)y_1(j)}{y_4^2(i)} \tag{27a}$$

$$y_{12} = \frac{z_3(i) + 2z_2(i)y_1(j)}{y_4^2(i)} \tag{27b}$$

$$y_{13} = -\frac{z_2(i)}{y_4^2(i)} \tag{27c}$$

$$y_{14} = \frac{z_1(i)}{y_4^2(i)} \tag{27d}$$

where z denotes the Cartesian state vector defined in Eq. (1). To simplify the analysis, the x axis of the Cartesian reference system (x,y) shown in Fig. 1 is chosen along the line of sight at the time t_j. In this reference frame, one has

$$y_3(j) = 0 \tag{28}$$

$$z_1(j) = y_4(j) \tag{29}$$

From Eq. (24), it follows immediately that $w_{15}(i,j)$ vanishes for all $i > j$ if

$$y_6(j) = 0 \tag{30}$$

Note that, because of Eq. (9), Eq. (30) implies that $y_6(i) = 0$ for all $i > j$. Hence, the target heading y_5 is unobservable for zero target maneuver. A second condition for w_{15} to vanish that results from Eq. (24) is

$$\mathrm{tg} y_5(j) = \frac{z_2(i)}{z_1(i)} \equiv \mathrm{tg} y_3(i) \quad \text{for all } i > j \tag{31}$$

Obviously, Eq. (31) is satisfied only for constant bearing angle. In this case, the target heading angle is unobservable if the velocity v_T is directed along the line of sight (LOS).

Some qualitative statements about the observability of y_5 can be obtained by assuming that the range y_4 is large compared to the relative displacement of observer and target. In intercept scenarios, this situation occurs during the initial observation period. Because of the selection of the Cartesian reference system according to Eq. (28), one has, at t_j,

$$z_2(j) = 0 \tag{32}$$

By assumption, at time t_i,

$$z_2(i) \ll z_1(i) \tag{33}$$

and one obtains approximately

$$w_{15}(i,j) \approx -\frac{1}{y_4^2(i)} T_{ij} v_T z_1(i) y_6(j) \sin y_5(j) \tag{34}$$

Thus, the observability of y_5 is maximal if the target heading is normal to the LOS ($|y_5(j)| = \pi/2$) and minimal if it is parallel to the LOS ($y_5(j) = 0$). This is intuitively clear because the target motion along the LOS does not influence the bearing dynamics directly but only in an *indirect* way via the range dynamics.

In summary, for nonzero target acceleration, the observability of the target heading is minimal (maximal) if the target velocity is directed parallel (normal) to the LOS. The target heading is unobservable for nonmaneuvering target.

To analyze the sensitivity w_{16}, the coefficients β_1 and β_2, according to Eq. (26), will now be approximated by again using the assumption of small relative displacements compared to the range. In addition to Eqs. (32) and (33), the following relations hold:

$$T_{ij} z_3(i) \ll R(i) \tag{35}$$

$$T_{ij} y_1(i) \ll 1 \tag{36}$$

$$T_{ij} y_6(i) \ll 1 \tag{37}$$

$$z_4(i) T_{ij} \approx z_1(i) y_1(i) T_{ij} \approx z_2(i) \tag{38}$$

Note that Eq. (36) is typically satisfied in intercept scenarios with bearing-rate feedback such as PN. Equation (29) with Eq. (27d) yields

$$|y_{14}| \approx \frac{1}{y_4(i)} \tag{39}$$

Equations (35) and (36) in Eq. (27b) and Eq. (33) in Eq. (27c) render

$$T_{ij} |y_{12}| \ll \frac{1}{y_4(i)} \tag{40}$$

$$|y_{13}| \ll \frac{1}{y_4(i)} \tag{41}$$

With Eqs. (39–41), it follows that

$$|\beta_2| \approx |y_{14}| \approx \frac{1}{y_4(i)} \tag{42}$$

Equation (38) in Eqs. (27a) and (27c) yields

$$y_{11} T_{ij} = c(T_{ij}) y_{13} \tag{43}$$

where

$$c(T_{ij}) = \mathcal{O}(1) \tag{44}$$

With Eqs. (43) and (44), β_1 becomes

$$\beta_1 = y_{14}\{[1 + c(T_{ij})]\varepsilon(i) + T_{ij} y_6(j)\} \tag{45}$$

with

$$\varepsilon(i) = \frac{y_{13}}{y_{14}} = \frac{z_2(i)}{z_1(i)} = \operatorname{tg}\varphi(i) \tag{46}$$

Using Eq. (33), one obtains, from Eq. (46),

$$\varepsilon(i) \approx \varphi(i) \ll 1 \tag{47}$$

and, with Eq. (44), it follows that

$$[1 + c(T_{ij})]\varepsilon(i) \ll 1 \tag{48}$$

Substitution of Eqs. (37), (39), and (48) into Eq. (45) and comparison with Eq. (42) yield

$$|\beta_2| \gg |\beta_1| \tag{49}$$

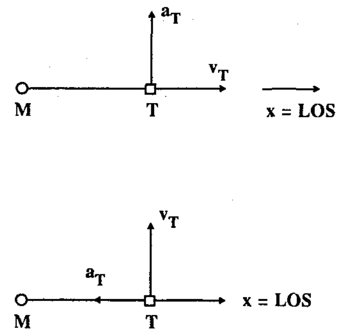

Fig. 3 Minimum (maximum) observability of y_5 (y_6); maximum (minimum) observability of y_5 (y_6).

Therefore, it may be concluded from Eq. (25) that $|w_{16}|$ is maximal for $y_5(j) = 0$ and minimal for $|y_5(j)| = \pi/2$. In other words, the observability of y_6 is maximal if $\boldsymbol{a}_T$ is directed normal to the LOS and minimal if $\boldsymbol{a}_T$ is directed along the LOS. Comparison with the results obtained for y_5 reveals that the target heading with maximum observability of y_6 results in minimum observability of y_5, and vice versa (Fig. 3). Moreover, it is evident from Eqs. (25) and (42) that the observability of y_6 is practically independent of the value of y_6 for $y_5(j) = 0$. In contrast, if $|y_5(j)| = \pi/2$, the observability depends strongly on the target maneuver. The conclusions from Eqs. (25) and (45) for this case are:

1) The observability of y_6 is minimal for zero target maneuver, i.e., $y_6(j) = 0$.

2) The observability of y_6 is maximal if

$$y_6(j) \gg \varphi(i)/T_{ij} \tag{50}$$

i.e., the observability of y_6 is maximal if the target heading rate is much larger than the average bearing rate.

The results of the observability analysis may be summarized as follows: The tracking filter is "blind" toward target maneuvers along the LOS, and the target heading is unobservable for zero target acceleration. Target maneuvers can be tracked only if they influence the bearing rate directly, i.e., if they have a component normal to the LOS. Clearly, the "blindness" with respect to motions along the LOS could be avoided by range or range-rate measurements. However, according to the preconditions of this study, these measurements are not available.

The analysis of the observability of target maneuvers via bearing-only measurements may be carried out by analyzing the sensitivities w_{35} and w_{36} as defined in Eq. (22). The results are reported in Ref. 19. Because the bearing angle is the integral of the bearing rate, the conclusions obtained for bearing-rate-only measurements apply to bearing-only measurements as well. In addition, the investigation of a singularly perturbed version of the tracking equations suggests that target maneuvers are practically unobservable via bearing-only measurements in scenarios involving low average bearing rates.[20] These scenarios are typical when guidance laws of the type PN are implemented.

IV. Simulation Results

The diagonal elements p_{ii} of the covariance matrix P are the filter's estimates of the error variance associated with the states y_i. According to Eqs. (7) and (9), the subsystem (y_5, y_6) is unstable. Hence, any estimation error in these states can be reduced only via the filter's update equations but not by merely solving the propagation equations. Because only the observable states are influenced by new measurements, the observability of the states y_5 and y_6 is indicated by the behavior of p_{55} and p_{66}, respectively. These variances will decrease only if y_5 and y_6 are observable.

The results of the observability analysis will be demonstrated by comparing the variance histories $p_{55}(t)$ and $p_{66}(t)$ for different intercept geometries. The variances are computed by solving the covariance equations of the tracking filter along precomputed deterministic flight paths. The intercept geometry is characterized by the initial conditions according to Fig. 4.

Consider first the following scenario (see Fig. 4).

Scenario 1:

$\gamma_{T0} = 0.25\pi$	(initial target heading)
$\gamma_0 = 0$	(initial missile heading)
$a_T = 0$	(target acceleration)
$R_0 = 5$ km	(initial range)

The flight paths associated with scenario 1 are shown in Fig. 5. The filtering results are depicted in Fig. 6.

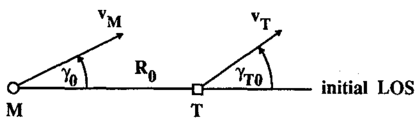

Fig. 4 Initial intercept geometry.

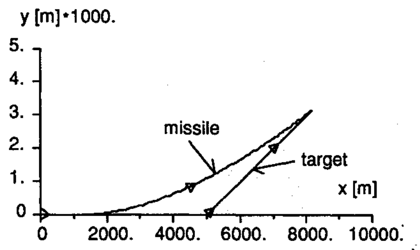

Fig. 5 Missile and target flight path in scenario 1.

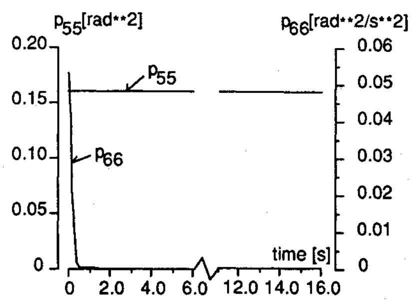

Fig. 6 Error variances of y_5 and y_6 in scenario 1.

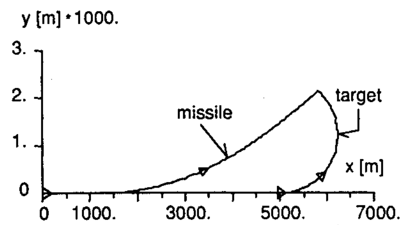

Fig. 7 Missile and target flight path in scenario 2.

It was shown that the target heading is unobservable for a nonmaneuvering target. This is reflected in the p_{55} history in Fig. 6: p_{55} remains constant throughout the observation interval. In contrast, the target heading rate is observable as indicated by the fast decay of p_{66}.

The initial observability of y_5 and y_6 depends on the initial target heading γ_{T0}. This is seen by comparing the histories of p_{55} and p_{66} in the following scenarios (see Fig. 4).

Scenario 2 (Figs. 7 and 9):

$$\gamma_{T0} = 0, \quad \gamma_0 = 0, \quad a_T = 0, \quad R_0 = 5 \text{ km}$$

Scenario 3 (Figs. 8–11):

$$\gamma_{T0} = 0.5\pi, \quad \gamma_0 = 0, \quad a_T = 6g, \quad R_0 = 5.5 \text{ km}$$

According to Sec. III, the initial observability of y_5 is minimal in scenario 2 and maximal in scenario 3. This is reflected by the initial behavior of p_{55} in Fig. 9: Increasing p_{55} indicates low observability of y_5 in scenario 2, decreasing p_{55} indicates high observability of y_5 in scenario 3. On the other hand, the initial observability of y_6 is maximal in scenario 2 and minimal in scenario 3, which is confirmed in Fig. 9.

The observability analysis has also shown that, for scenario 3, where the initial target acceleration is directed along the initial LOS, the observability of y_6 vanishes for zero target maneuver. This is demonstrated by solving the filter equations based on the linearization around the estimated trajectory instead of the precomputed trajectory. If the estimated target acceleration (or target heading rate) is low, the sensitivity w_{16}

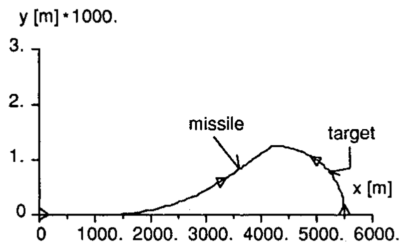

Fig. 8 Missile and target flight path in scenario 3.

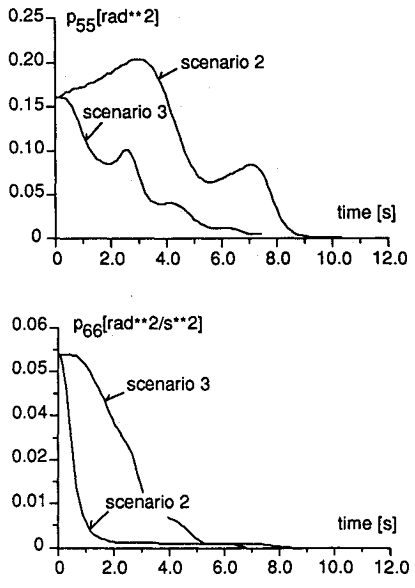

Fig. 9 Error variances of y_5 and y_6 in scenarios 2 and 3.

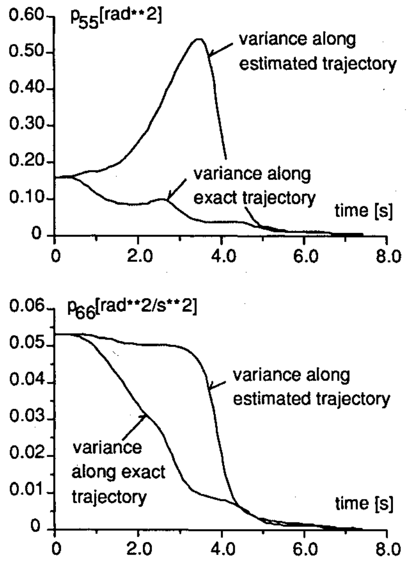

Fig. 10 Error variances of y_5 and y_6 in scenario 3.

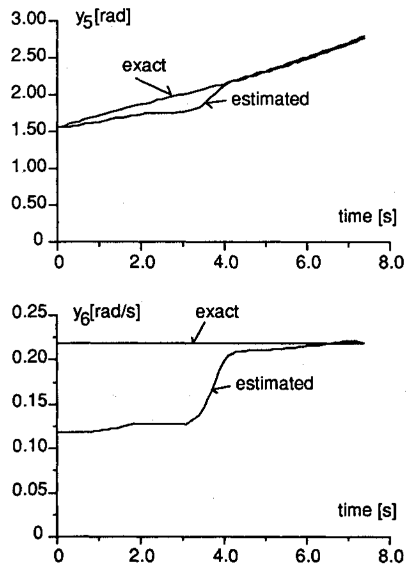

Fig. 11 Exact and estimated trajectories of y_5 and y_6 in scenario 3.

is low according to Eqs. (25) and (45). Consequently, the variance p_{66} will decrease only slowly on the estimated target trajectory. This is evident from the comparison of the p_{66} histories shown in Fig. 10. On the estimated trajectory, the initial estimate of y_6 was half the exact value (see Fig. 11). The resulting low acceleration profile during the first half of the estimated trajectory also results in low observability of y_5 as evident from Figs. 10 and 11. As soon as the estimated acceleration level rises, a drastic increase in the observability of both y_5 and y_6 accompanied by fast convergence of the estimates occurs.

V. Conclusions

The problem of tracking maneuvering targets via bearing-only and bearing-rate-only measurements has been discussed. The constant-speed target model (CSTM) was used to describe target maneuvers in short-range intercept scenarios. The basic tracking algorithm is the extended Kalman filter formulated in polar coordinates. An observability analysis has shown that target maneuvers along the line of sight are practically unobservable via bearing-(rate-)only observations. As a consequence, guidance laws that require information about radial target maneuvers are implementable only if range or range-rate measurements are available. On the other hand, the lack of observability of radial target maneuvers does not necessarily affect the missile guidance law if it is based on information about the target maneuver normal to the line of sight only. Extensions of proportional navigation are usually of this type.

Simulations confirmed the results of the observability analysis. It was shown that the behavior of the tracking filter is highly dependent on the intercept geometry. This is a direct consequence of the fact that only target maneuvers normal to the line of sight are observable. Moreover, since, in practice, the tracking algorithm evaluates the covariance matrix along the *estimated* state trajectory, the initial state estimates have a strong influence on observability. As an example, it was demonstrated that initial target acceleration estimates that are too low result in a considerable loss of observability that may even result in the instability of the filter. It becomes evident that an initial guess of zero target acceleration may be the worst possible value to initialize the filter with. The observability analysis provides a guideline on how to initialize the tracking filter in different intercept geometries in order to avoid instability because of the loss of observability right at initial time.

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